

4º $\vec{u} = (1, 0, 1)$ $\vec{v} = (0, 2, 1)$ $\vec{w} = (m, 1, n)$

a) ¿m y n?

Si \vec{u}, \vec{v} y \vec{w} son linealmente dependientes, $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ m & 1 & n \end{vmatrix} = 0$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ m & 1 & n \end{vmatrix} = 0 = 2n + 0 + 0 - 2m - 1 + 0 = 0$ $2n - 2m - 1 = 0$

Si $\vec{w} \perp \vec{u} \rightarrow \vec{u} \cdot \vec{w} = 0$ $(1, 0, 1) \cdot (m, 1, n) = 0 = \boxed{m + n = 0}$

$m = -n$ $2n + 2n - 1 = 0$ $4n = 1$ $n = \frac{1}{4}$ $m = -\frac{1}{4}$

b) $n=1$ ¿m?

$V_{\text{tetraedro}} = \frac{1}{6} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ m & 1 & 1 \end{vmatrix} = 10 u^3$; $\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ m & 1 & 1 \end{vmatrix} = 60$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ m & 1 & 1 \end{vmatrix} = 2 + 0 + 0 - 2m - 1 + 0 = 60$

$1 - 2m = 60$

$2m = -59$

$m = -\frac{59}{2}$