MAT II OPCIÓN B 13/06/2017 ANDALUCÍA

2º $\boxed{\displaystyle\int_1^{16} \frac{dx}{\sqrt{x}+\sqrt[4]{x}}} = 4\cdot\left[\frac{\sqrt{x}}{2} - \sqrt[4]{x} + \ln\left(\sqrt[4]{x}+1\right)\right]_1^{16} = I^*$

$\underline{t = \sqrt[4]{x}} = x^{1/4} \qquad dt = \frac{1}{4}\cdot x^{-3/4}\cdot dx \;;\; \underline{dx} = 4\,dt\cdot x^{3/4} = 4\cdot\sqrt[4]{x^3}\cdot dt = \underline{4t^3 dt}$

$t^4 = \left(x^{1/4}\right)^4 \;;\; t^4 = x \qquad \underline{\sqrt{x}} = \underline{t^2}$

$$I = \int \frac{dx}{\sqrt{x}+\sqrt[4]{x}} = \int \frac{4t^3 dt}{t^2+t} = 4\int \frac{t^2}{t+1}\,dt = 4\int\left(t-1+\frac{1}{t+1}\right)dt = 4\left(\frac{t^2}{2} - t + \ln(t+1)\right) = \ldots$$

$$\begin{array}{r|l} \cancel{t^2} & t+1 \\ \cancel{-t^2}-t & t-1 \\ \hline -\cancel{t} & \\ \cancel{+t}+1 & \\ \hline 1 & \end{array} \qquad \frac{t^2}{t+1} = t-1+\frac{1}{t+1}$$

$$\ldots = I = 4\left(\frac{\sqrt{x}}{2} - \sqrt[4]{x} + \ln\left(\sqrt[4]{x}+1\right)\right) + C$$

$$I^* = 4\cdot\left[\frac{\sqrt{16}}{2} - \sqrt[4]{16} + \ln\left(\sqrt[4]{16}+1\right)\right] - 4\cdot\left[\frac{\sqrt{1}}{2} - \sqrt[4]{1} + \ln\left(\sqrt[4]{1}+1\right)\right] =$$

$$= 4\cdot\left(\frac{4}{2} - 2 + \ln(2+1)\right) - 4\cdot\left(\frac{1}{2} - 1 + \ln(1+1)\right) = 4\left(\cancel{2}-\cancel{2}+\ln 3\right) - 4\left(\frac{1}{2} - 1 + \ln 2\right) =$$

$$= 4\cdot\ln 3 - 4\left(-\frac{1}{2} + \ln 2\right) = 4\cdot\ln 3 + 2 - 4\ln 2 = \boxed{2 + 4\left(\ln 3 - \ln 2\right)}$$

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